4.2 - Adding, Subtracting, and Multiplying Polynomials

Warmup

1. Fifth term of $(x+y)^7$

$$\frac{7!}{3!4!}x^3y^4 = 35x^3y^4$$

2. Fourth term of $(2x+3y)^9$

$$\frac{9!}{6!3!}(2x)^6(3y)^3 = 145{,}152x^6y^3$$

3. Seventh term of $(x-y)^{15}$

$$\frac{15!}{9!6!}x^9(-y)^6 = 5{,}005x^9y^6$$

4. Fifth term of $(x-2)^{10}$

$$\frac{10!}{6!4!}x^6(-2)^4 = 3{,}360x^6$$

5. Sixth term of $(2m+3n)^{12}$

$$\frac{12!}{7!5!}(2m)^7(3n)^5 = 24{,}634{,}368m^7n^5$$

6. Eighth term of $(3a+5b)^{11}$

$$\frac{11!}{4!7!}(3a)^4(5b)^7 = 2{,}088{,}281{,}250a^4b^7$$

4.3 - Dividing Polynomials

Division of Polynomials

When you divide a polynomial f(x) by a nonzero polynomial divisor d(x), you get a quotient polynomial q(x) and a remainder polynomial r(x).

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

4.3 - Dividing Polynomials

Synthetic Division

There is a shortcut for dividing polynomials by binomials of the form (x - k).

Use synthetic division on: $$\frac{2x^3 - x - 7}{x + 3}$$

$$2x^2 - 6x + 17 - \frac{58}{x + 3}$$

4.3 - Dividing Polynomials

Synthetic Division

There is a shortcut for dividing polynomials by binomials of the form (x-k).

Use synthetic division on: $$\frac{2x^3 - x - 7}{x + 3}$$

4.3 - Dividing Polynomials

Synthetic Division

Divide $x^4 - 2x^3 + 13x - 6$ by $x + 2$

$$x^3 - 4x^2 + 8x - 3 + \frac{0}{x+2}$$

4.3 - Dividing Polynomials

Practice - Synthetic Division

1. $\dfrac{t^4 + 5t^3 - 2t - 7}{t + 5}$

$t^3 - 2 + \dfrac{3}{t + 5}$

2. $\dfrac{2u^4 - 5u^3 - 12u^2 + 2u - 8}{u - 4}$

$2u^3 + 3u^2 + 2$

4.3 - Dividing Polynomials

Practice - Synthetic Division

1. $(9x^3 - 16x - 18x^2 + 32)/(x - 2)$ $\quad 9x^2 - 16$

2. $(-x^3 + 75x - 250)/(x + 10)$ $\quad -x^2 + 10x - 25$

3. $(-3x^3 + 5x^2 - x + i)/(x - i)$

$$-3x^2 + (5 - 3i)x + (2 + 5i) + \frac{-5 + 3i}{x - i}$$

4.3 - Dividing Polynomials

Polynomial Long Division

$$\frac{x^3 - x^2 - 2x + 8}{x - 1}$$

$$x^2 - 2 + \frac{6}{x - 1}$$

$$\frac{34x - 16 + 15x^2}{5x - 2}$$

$$3x + 8$$

4.3 - Dividing Polynomials

Practice - Polynomial Long Division

1. $\dfrac{8x^2 + 6x + 3}{4x + 1}$ $\quad 2x + 1 + \dfrac{2}{4x + 1}$

2. $\dfrac{2a^3 + 5}{a - 3}$ $\quad 2a^2 + 6a + 18 + \dfrac{59}{a - 3}$

4.3 - Dividing Polynomials

The Remainder Theorem

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$ **Polynomial division**

$$\frac{f(x)}{x-k} = q(x) + \frac{r(x)}{x-k}$$ **Substitute x-k for d(x)**

$$f(x) = (x-k)q(x) + r(x)$$ **Multiply by x-k**

$$f(k) = (k-k)q(k) + r(k)$$ **Substitute k for x**

$$f(k) = r(k)$$ **Simplify**

4.3 - Dividing Polynomials

The Remainder Theorem

If a polynomial f(x) is divided by x-k, then the remainder is r = f(k).

Use Synthetic Substitution to evaluate

$$f(x) = 5x^4 + 2x^3 - 20x - 6;\ \ x = 2$$

$$f(2) = 50$$

4.3 - Dividing Polynomials

Practice - Synthetic Substitution

1. $f(x) = x^4 - 14x^2 + 5x - 3$

   $f(-4) =$ 9

2. $f(x) = x^3 + 2x^2 - x + 4$

   $f(-5) =$ -66

4.4 - Factoring Polynomials

The Factor Theorem

A polynomial f(x) has a factor x-k if and only if f(k) = 0.

$$\frac{f(x)}{x-k} = q(x) + \frac{0}{d(x)} = q(x)$$

$$f(x) = (x-k)q(x)$$

4.5 - Solving Polynomial Equations

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

For example $x^3 - 8x^2 + 11x + 20 = 0$

Possible rational roots

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$$

Actual roots

$$x = \{-1, 4, 5\}$$